Title: Focusing on Parabolas

Brief Overview:

Algebra II students will develop an understanding of parabolas based on the focus–directrix definition. Students will construct parabolas, derive their equations, and solve problems by applying those equations. Students will discover important properties of the parabola and its focus, then investigate applications of those properties.

NCTM Content Standards/MD Content Standards:

- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Use symbolic algebra to represent and explain mathematical relationships
- Use symbolic expressions to represent relationships arising from various contexts
- Model and interpret real–world situations using the language of mathematics and appropriate technology
- Determine and interpret information from models of simple conic sections
- Describe circles, ellipses, parabolas, and hyperbolas as loci of points

Grade/Level:

Grades 9 – 12; Algebra II and Pre–calculus

Duration/Length:

Four 75 – 90 periods

Student Outcomes:

Students will:

- Define a parabola, based on its focus and directrix
- Graph parabolas and describe their properties
- Write equations of parabolas from given information
- Describe applications of parabolas

Materials and Resources:

- Calculators
- Rope
- Mini–whiteboards
- Whiteboard markers
- Erasers

- Patty paper or wax paper
- Computers with Internet access
- Worksheets
    - Distance and Equidistance Warm Up
    - Graphing Equidistant Points
    - Parabola Basics Homework
    - A Second Look at Parabolas
    - Plotting Parabolas Homework
    - Polishing Up Parabolas Warm Up
    - Goin' Round with Parabolas
    - Go Around Again!
    - Making a U-Turn Homework
    - One More Look at Parabolas
    - Parabolas Extension and Challenge Problems
    - Web Investigation – Focusing on Parabolas
    - Focusing on Parabolas Quiz

Development/Procedures:

Lesson 1

Pre–assessment – Assign the warm–up, "Distance and Equidistance". This assignment will assess student recall of the distance formula. Take time to develop the distance formula using the Pythagorean Theorem, if the students are not already familiar with this proof.

Launch – Prior to class, knot a rope in one–foot increments. In an open area with a tile floor, mark a line to be the directrix. Mark a point 2 feet away from the line to be the focus. After the warm–up, invite a student to stand at the focus, *F*. Ask a second student, *D*, to walk along the directrix. Use the remaining students in the class to form a human parabola. To plot the first point of the parabola, a third student (*P*) will hold the midpoint of the rope. Students *F* and *D* will hold the knots 1 foot away on each side of *P*; make sure the rope is taut and is perpendicular to the directrix. Say to the students, "Where *P* is now standing represents one point equidistant from the focus and directrix." Student *P* stays in place, and another student (*Q*) holds the midpoint of the rope. Students *F* and *D* now hold the knots 2 feet away on each side of *Q*. Students *D* and *Q* should move until the rope is again taut and perpendicular to the directrix. Say to students, "It is important for the rope to always be taut and perpendicular to the directrix. Where *Q* is now standing represents another point equidistant from the focus and directrix." There are two possible locations for *Q*, so have a second student stand at the other. Continue until enough students are standing at points on the curve to predict its shape – a parabola!

Teacher Facilitation – Transition into the mathematics using Part A from the packet, "Graphing Equidistant Points." Make connections to the human parabola as the students identify points on the grid that are equidistant from the focus and directix. Remind the students that the segment drawn from the point along the parabola to the directrix must be perpendicular to the directrix.

Student Application – Divide the class into pairs or small groups. Have the students complete the remaining parts of the packet, with varying amounts of teacher guidance, depending on the level of students. Monitor student progress, answering questions as they arise. Keep the assistance to a minimum, allowing for student exploration and discovery. When necessary, invite students to the board to present solutions to different sections.

Assign Homework – Parabola Basics

Lesson 2

Pre–assessment – Students will complete the Review section from "A Second Look at Parabolas" to check their understanding of concepts from Day 1. Also, review the homework from the previous section and discuss any questions.

Launch – Distribute the "Folding Parabolas" worksheet and a piece of patty paper to each student. This is a hands–on activity to reinforce the focus-directrix definition of a parabola. Note that this simulation can be done on a construction program such as Geometer's Sketchpad. There are also Applets available online to simulate the process and produce a more accurate graph.

Teacher Facilitation – Return to the sheet "A Second Look at Parabolas." Teacher will lead a discussion of the questions from the Predict section, then model the first example from the Apply section.

Student Application – Students will complete and check the remaining two problems from “A Second Look at Parabolas.”

Embedded Assessment – Distribute response boards, preferably ones which are two–sided (with a blank side and a coordinate grid side). In this activity, students will practice writing equations of parabolas with vertex at the origin, given particular criteria. Using an overhead project, display “Response Board Practice”, and work through the two examples with the class. Display the practice set of exercises, one exercise at a time. Cover the solutions with a sticky note. Have the students can work with a partner, and then individually, to write the equation to match each criteria. The students use the response boards to display their equations, and graph the parabolas. Remove the sticky note to show the students the equations, so they can verify their solution. The last exercise should generate a lot of discussion! Ask the students, “Can the focus lie on the directrix?” Explain to the students that letting $p = 0$ produces the equation $x^2 = 0$, which becomes $x = 0$. This is a line and not a parabola. Ask, “Does this line satisfy the ‘equidistant’ definition presented at the beginning of class?”

Assign Homework – Plotting Parabolas

Lesson 3

Pre–assessment – The students will complete the “Polishing Up Parabolas” warm up to review the content from the previous lesson. Monitor the students’ work to clarify any misconceptions. The students will then explore the equation for parabolas as the vertex is moved off of the origin. The students will be introduced to the equations $(y-k)^2 = 4p(x-h)$ and $(x-h)^2 = 4p(y-k)$. Review the exercises from the warm-up as well as the homework from the previous lesson, clarifying any mistakes.

Teacher Facilitation – Display the parabolic equations $(y-k)^2 = 4p(x-h)$ and $(x-h)^2 = 4p(y-k)$ on the board. Model examples of how to graph each type of parabola for the student. After the first round table activity, model examples of how to write a parabola given criteria such as the vertex & focus.

Student Application – Divide the class into groups of three for the round table activity, “Goin’ Round with Parabolas.” In this activity, the students will practice graphing parabolas, identifying the focus, vertex, directrix, and latus rectum. Peer assessment will help guide and check student work.

Following the “Goin’ Round with Parabolas” the students will participate in a second round table activity, “Go Around Again!” The second round table activity has the students practice writing parabolic equations given a set of conditions.

Embedded Assessment – Assess student progress during the round table activities, correcting any errors and clarifying any misconceptions. Collect the sheets at the end of the activity.

Assign Homework – Making a U-Turn

Lesson 4

Pre–assessment – Begin the lesson by going over the homework from the previous lesson. After checking the HW, students will have a choice of two activities, depending on how successful they were with the homework and their comfort level with the material.

Reteaching /Extension –

- Students who missed a lot of problems on the homework or would like extra practice with the material should choose the reteaching activity.
- Students who got most of the homework correct and feel comfortable with the topic should work on the sheet “Parabolas – Extension and Challenge Problems” in pairs or small groups.
- For the last 30 minutes of the block, all students will complete the “Web Investigation – Focusing on Parabolas” to learn about applications of parabolas and their focus.

Students will turn in this sheet at the end of class for a grade.

Summative Assessment:

There will be a summative assessment (quiz) on the following day.

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Distance and Equidistance Name: ________________________

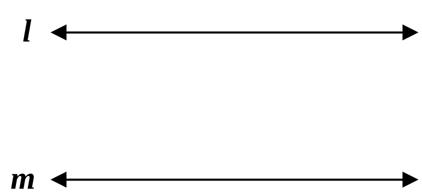
Warm Up Date: _________________________

1. Given the points (x_1, y_1) and (x_2, y_2), identify the formula used to calculate the distance between the two points. Where does this formula come from?

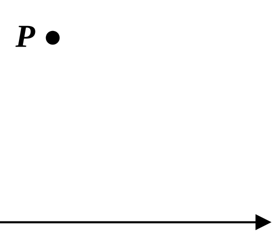
2. Recall the definition of a *locus* (a set of points that satisfy certain conditions). Sketch and describe each of the following *loci*.

   a. The set of points equidistant from two points, *P* and *Q*.

   b. The set of points equidistant from two parallel lines, *l* and *m*.



3. Make a prediction: Given a point *P* and a line *l*, what would the set of points equidistant from *P* and *l* look like? Why?



Distance and Equidistance
Warm Up

Name: __ANSWER KEY_________
Date: __________________________

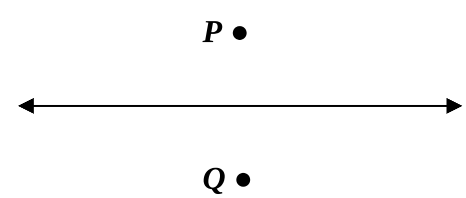
1. Given the points (x_1, y_1) and (x_2, y_2), identify the formula used to calculate the distance between the two points. Where does this formula come from?

   Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

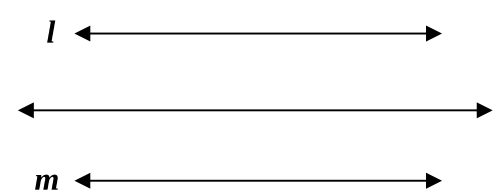
   It is derived from the Pythagorean Theorem.

2. Recall the definition of a *locus* (a set of points that satisfy certain conditions). Sketch and describe each of the following *loci*.

   a. The set of points equidistant from two points, *P* and *Q*.
   The perpendicular bisector of $\overline{PQ}$

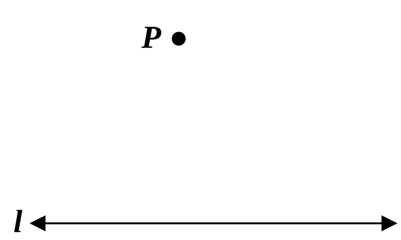


   b. The set of points equidistant from two parallel lines, *l* and *m*.
   A line parallel to and halfway between *l* and *m*.



4. Make a prediction: Given a point *P* and a line *l*, what would the set of points equidistant from *P* and *l* look like? Why?

   Answers will vary; make sure students justify their responses.



Graphing Equidistant Points

Name: ______________________

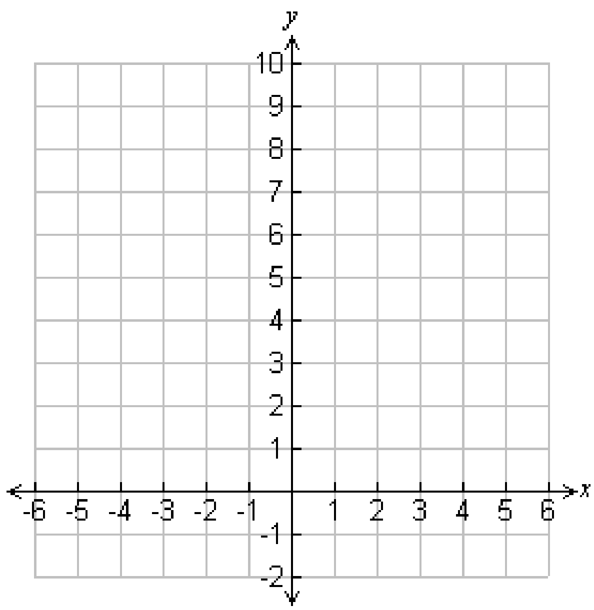
Date: ______________________

What does the set of points equidistant from a point and a line look like? The following activity will help determine the shape of such a graph, derive its equation and explore its properties.

A. Determining the Graph

1.

- On the graph, plot the point P(0,1) and the line $y = -1$. Label the line *l*.
- Find one point in the plane that is equidistant from P and *l*. Label this point on the graph below.



2.

- Find several ordered pairs that are equidistant from P and *l*. Plot these points.
- Verify that the distances are the same using the distance formula.

3.

- Make a prediction about the type of graph formed by the points equidistant from *P* and *l* .
- What is the equation of this graph?

B. Equations of Parabolas

The equation of a parabola can be written in the following forms:

- Standard form: $y = ax^2 + bx + c$
- Vertex form: $y = a(x - h)^2 + k$

When studying parabolas as sets of equidistant points, the equation is written in another form.

1. Start by writing a definition based on the results from part A.

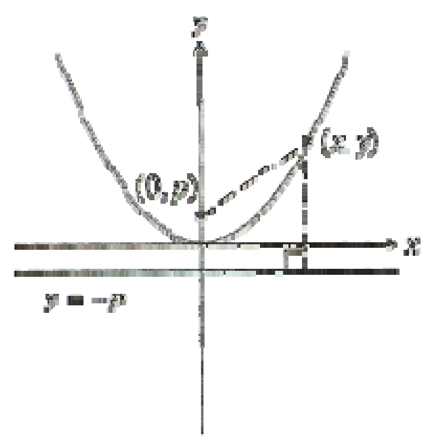
____

____

____

____

The equation for a parabola can be derived from this definition. We begin with parabolas that open vertically whose vertex is the origin.



2. Let the focus of a parabola be the point $(0, p)$. Why is the equation of the directrix $y = -p$?

____

____

3. Let (x, y) be a point on the parabola.

- Write an expression representing the distance from (x, y) to the focus $(0, p)$:

____

- Write an expression representing the distance from (x, y) to the directrix:

____

4. Derive the equation for the parabola.

- Set the two expressions from exercise 3 equal to each other.

- Square both sides of the equation.

- Expand any expressions in parentheses.

- Simplify and solve for x^2.

- Thus, the equation of a parabola with focus (0, *p*), the vertex at the origin, and a directrix of $y = -p$ is:

C. Graphing a Parabola

To sketch a parabola, the vertex and at least one point on each side of the vertex are needed. There is a quick way to get such a pair of points by using a segment called the *latus rectum*. The latus rectum is the segment connecting two points of the parabola. It is parallel to the directrix and contains the focus. The latus rectum determines the how wide the parabola opens.

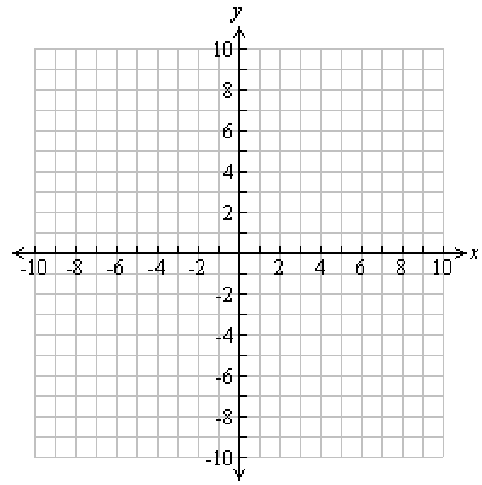
1. Determine the length of the latus rectum in any parabola using the following facts:

- The equation of the parabola is $x^2 = 4py$.
- The latus rectum lies on a horizontal line with the focus (0, p)

D. Practice

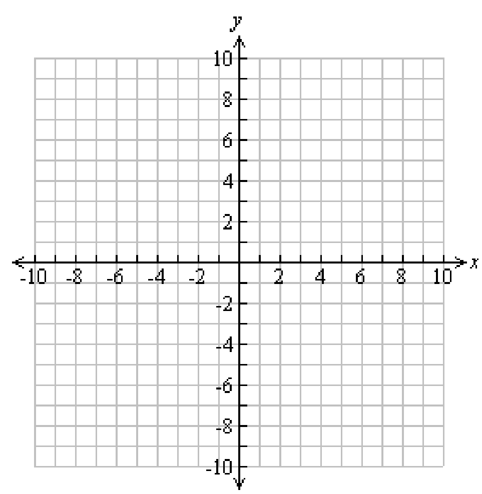
1. Change each equation to standard form, $x^2 = 4py$. Identify the focus, directrix, and length of latus rectum. Then sketch the graph.

a. $6y + x^2 = 0$



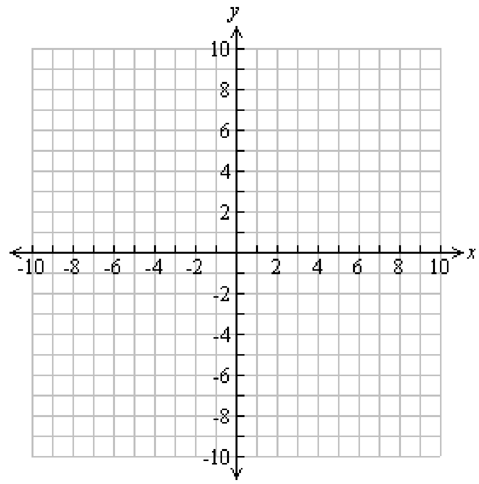
- Length of latus rectum: ___________
- Focus: ___________
- Directrix: ___________

b. $5x^2 = -60y$



- Length of latus rectum: ___________
- Focus: ___________
- Directrix: ___________

c. $8y - 3x^2 = 0$



- Length of latus rectum: __________

- Focus: __________

- Directrix: __________

Graphing Equidistant Points

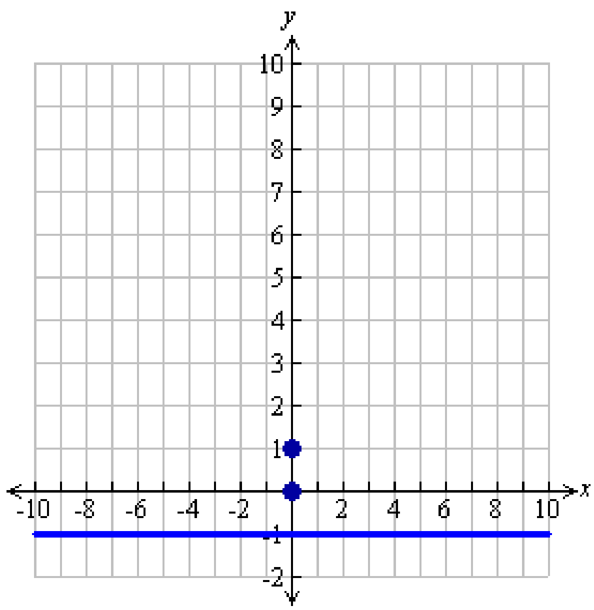
Name: __ANSWER KEY________
Date: __________________________

What does the set of points equidistant from a point and a line look like? The following activity, will help determine the shape of such a graph, derive its equation and explore its properties.

A. Determining the Graph

1.

- On the graph, plot the point P(0,1) and the line $y = -1$. Label the line *l*.
- Find one point in the plane that is equidistant from P and *l*. Label this point on the graph below.



2.

- Find several ordered pairs that are equidistant from P and *l*. Plot these points.
- Verify that the distances are the same using the distance formula.

(±2, 1) (±4, 4) (±6, 9)

3.

- Make a prediction about the type of graph formed by the points equidistant from *P* and *l* .
- What is the equation of this graph?

The graph is a parabola, $y = \dfrac{x^2}{4}$.

B. Equations of Parabolas

The equation of a parabola can be written in the following forms:

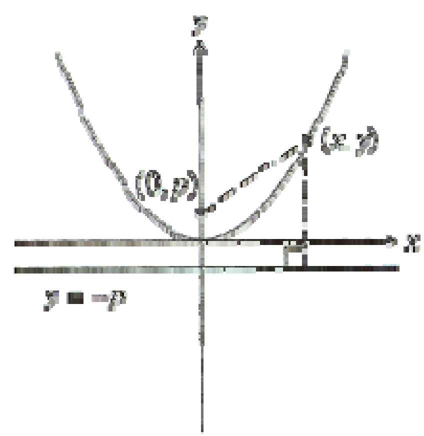
- ❖ Standard form: $y = ax^2 + bx + c$
- ❖ Vertex form: $y = a(x-h)^2 + k$

When studying parabolas as sets of equidistant points, the equation is written in another form.

1. Start by writing a definition based on the results from part A.

   A parabola is the set of points in a plane equidistant from a given line, the directrix, and a given point not on that line, the focus.

The equation for a parabola can be derived from this definition. We begin with parabolas that open vertically whose vertex is the origin.



2. Let the focus of a parabola be the point $(0, p)$. Why is the equation of the directrix $y = -p$?

The line must be p units below the origin so that the vertex is equidistant from the focus and directrix.

3. Let (x, y) be a point on the parabola.

- Write an expression representing the distance from (x, y) to the focus $(0, p)$:

  $\sqrt{x^2 + (y - p)^2}$

- Write an expression representing the distance from (x, y) to the directrix:

  $y + p$

4. Derive the equation for the parabola.

- Set the two expressions from exercise 3 equal to each other.

___ $\sqrt{x^2 + (y-p)^2} = y + p$

- Square both sides of the equation.

___ $x^2 + (y-p)^2 = (y+p)^2$

- Expand any expressions in parentheses.

___ $x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$

- Simplify and solve for x^2.

___ $x^2 = 4py$

- Thus, the equation of a parabola with focus (0, *p*), the vertex at the origin, and a directrix of $y = -p$ is:

$$\boxed{x^2 = 4py}$$

C. Graphing a Parabola

To sketch a parabola, the vertex and at least one point on each side of the vertex are needed. There is a quick way to get such a pair of points by using a segment called the *latus rectum*. The latus rectum is the segment connecting two points of the parabola, it is parallel to the directrix and contains the focus. The latus rectum determines the how wide the parabola opens.

1. Determine the length of the latus rectum in any parabola using the following facts:

- The equation of the parabola is $x^2 = 4py$.
- The latus rectum lies on a horizontal line with the focus $(0, p)$

The endpoints of the latus rectum are $(\pm x, p)$. Let $y = p$ in the equation, then

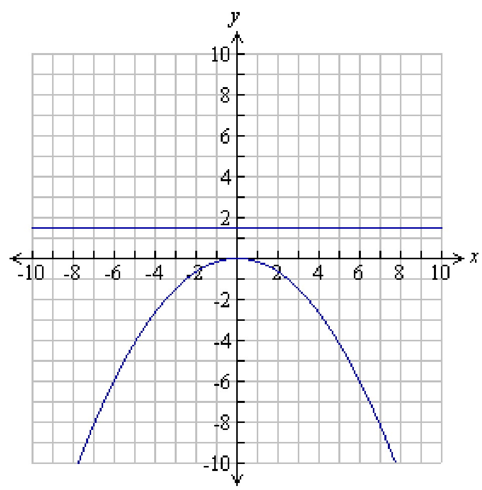
$$
\begin{aligned}
x^2 &= 4py \\
x^2 &= 4p(p) \\
x^2 &= 4p^2
\end{aligned}
$$

Thus, $x = \pm 2p$, so the length is $|2p - (-2p)| = |4p|$.

D. Practice

1. Change each equation to standard form, $x^2 = 4py$. Identify the focus, directrix, and length of latus rectum. Then sketch the graph.

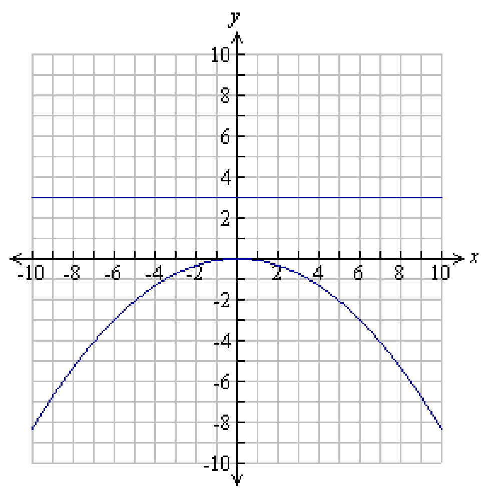
a. $6y + x^2 = 0$



$x^2 = -6y$, $p = -1.5$

- Length of latus rectum: ____6______
- Focus: __(0, –1.5)________
- Directrix: __$y = 1.5$________

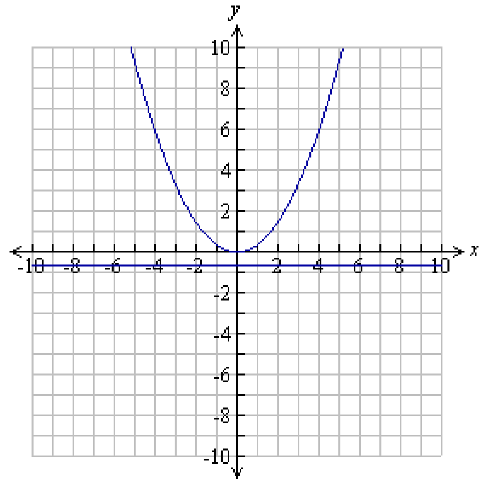
b. $5x^2 = -60y$



$x^2 = -12y$, $p = -3$

- Length of latus rectum: ___12_____
- Focus: ___(0, –3)_______
- Directrix: _____$y = 3$______

c. $8y - 3x^2 = 0$



$x^2 = \frac{8}{3}y,\ p = \frac{2}{3}$

- Length of latus rectum: ___$\underline{\frac{8}{3}}$______
- Focus: __$\underline{\left(0, \frac{2}{3}\right)}$________
- Directrix: ___$\underline{y = -\frac{2}{3}}$________

Parabola Basics Name: ________________________
Homework Date: _________________________

1. Complete each statement about parabolas.

   a. A parabola is the set of all points in a plane that are ________________________ from a given point, called the _______________, and a given line, called the _______________.

   b. The point on the parabola that is closest to the directrix and focus is the ______________.

   c. The equation of the parabola with vertex at the origin and focus (0, p) is _____________.

      The equation of its directrix is __________.

   d. There is a special line segment joining two points of a parabola. It contains the focus and is parallel to the directrix. This segment is called the _________________________. The length of this segment is _______.

   e. A vertical parabola with vertex at the origin has the ____________ as a line of symmetry.

2. Write the equation of the parabola with vertex at the origin that has the given property.

   Example **Focus is (0, 2.5)**

   Solution p = 2.5, so substituting into $x^2 = 4py$, we get

   $x^2 = 4(2.5)y \quad \rightarrow \quad$ $\underline{\mathbf{x^2 = 10y}}$

   a. Focus is (0, -3)

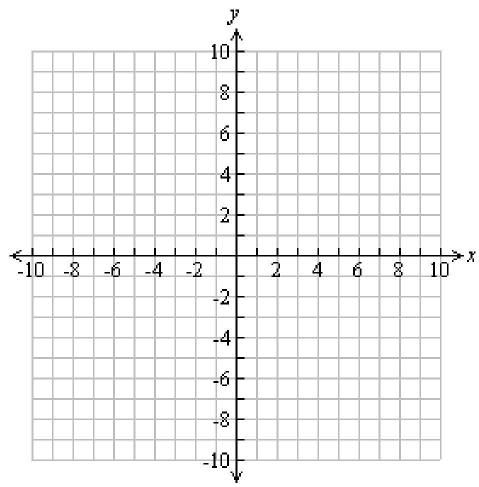
   b. Directrix is y = -5 (Remember, the equation of the directrix is y = -p)

c. Directrix is $y = \frac{2}{3}$

d. Parabola opens down, length of latus rectum is 15

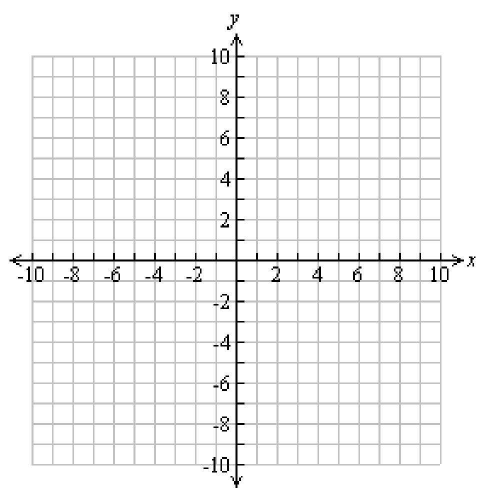
3. Change each equation to standard form, $x^2 = 4py$. Identify the focus, directrix, and length of latus rectum. Then sketch the graph.

a. $x^2 - 8y = 0$



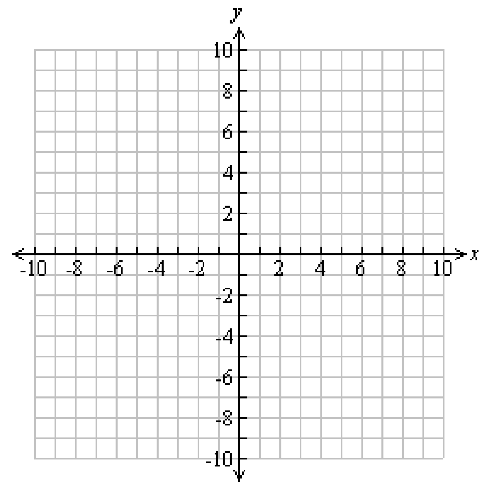
- Length of latus rectum: ___________
- Focus: ___________
- Directrix: ___________

b. $10y = -x^2$



- Length of latus rectum: ___________
- Focus: ___________
- Directrix: ___________

c. $5x^2 + 16y = 0$



- Length of latus rectum: __________
- Focus: __________
- Directrix: __________

Parabola Basics Name: __ANSWER KEY_________

Homework Date: __________________________

1. Complete each statement about parabolas.

   a. A parabola is the set of all points in a plane that are **equidistant** from a given point, called the **focus**, and a given line, called the **directrix**.

   d. The point on the parabola that is closest to the directrix and focus is the **vertex**.

   c. The equation of the parabola with vertex at the origin and focus (0, p) is $\mathbf{x^2 = 4py}$. The equation of its directrix is $\mathbf{y = -p}$.

   d. There is a special line segment joining two points of a parabola. It contains the focus and is parallel to the directrix. This segment is called the **latus rectum**. The length of this segment is **4p**.

   e. A vertical parabola with vertex at the origin has the **y-axis** as a line of symmetry.

2. Write the equation of the parabola with vertex at the origin that has the given property.

Example **Focus is (0, 2.5)**

Solution p = 2.5, so substituting into $x^2 = 4py$, we get

$x^2 = 4(2.5)y \rightarrow \mathbf{x^2 = 10y}$

a. Focus is (0, -3)

$\mathbf{x^2 = -12y}$

b. Directrix is y = -5 (Remember, the equation of the directrix is y = -p)

$\mathbf{x^2 = 20y}$

c. Directrix is y = ⅔

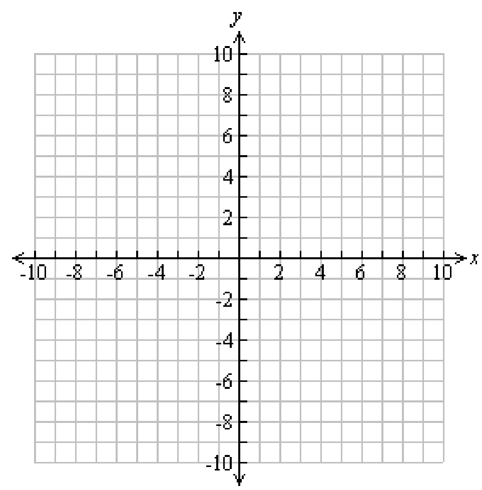
$\mathbf{x^2 = -8/3y}$

d. Parabola opens down, length of latus rectum is 15

$x^2 = -15y$

3. Change each equation to standard form, $x^2 = 4py$. Identify the focus, directrix, and length of latus rectum. Then sketch the graph.

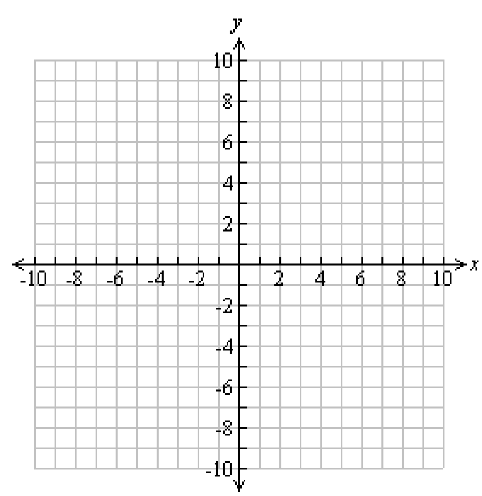
a. $x^2 - 8y = 0$



$x^2 = 8y$

- Length of latus rectum: __8___
- Focus: _(0, 2)__
- Directrix: _y = -2_

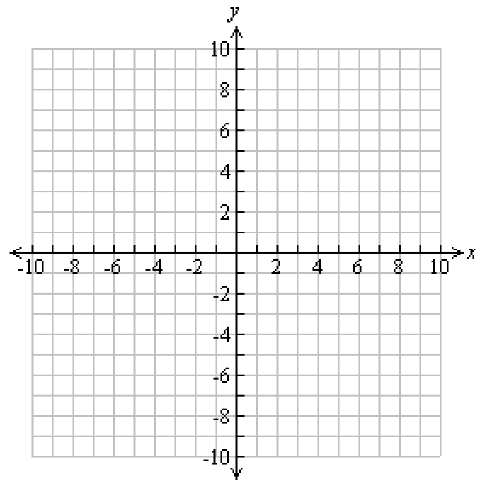
b. $10y = -x^2$



$x^2 = -10y$

- Length of latus rectum: _10__
- Focus: __(0, -2.5)___
- Directrix: _y = 2.5___

c. $5x^2 + 16y = 0$



x^2 = -3.2y

- Length of latus rectum: __3.2__
- Focus: __(0, -0.8)___
- Directrix: _y = 0.8___

A Second Look at Parabolas

Name: ________________________
Date: _________________________

Review

1. Yesterday, you learned that $x^2 = 4py$ is the equation of a parabola which opens vertically. How can you tell whether the parabola opens up or down?

2. What does the value p represent in the parabola's equation?

3. The latus rectum is a special segment associated with parabolas. Describe the properties of the latus rectum.

Predict

Take the equation of a vertical parabola and switch the x and y, so that the equation becomes $y^2 = 4px$. Make some predictions about this new equation.

1. How will the graph change?

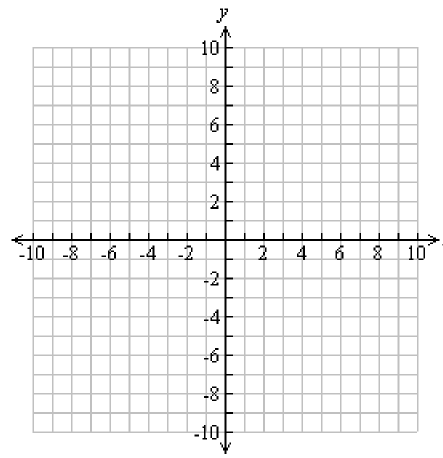
2. How will the sign of p affect the graph?

3. Will the equation $y^2 = 4px$ represent a function? Explain why or why not.

Apply

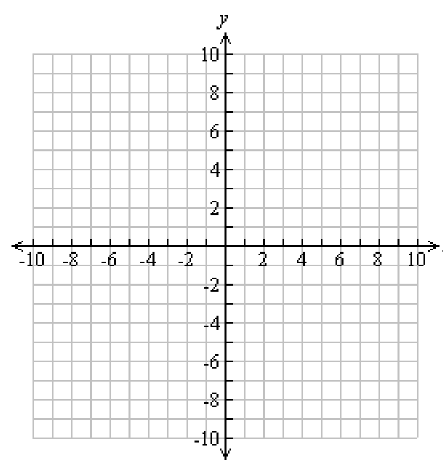
Change each equation to standard form, $y^2 = 4px$. Identify the focus, directrix, and length of latus rectum. Then sketch the graph.

1. $y^2 - 5x = 0$



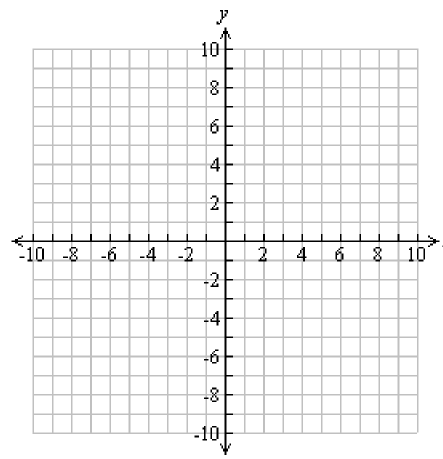
- Length of latus rectum: ___________
- Focus: ___________
- Directrix: ___________

2. $3y^2 = 36x$



- Length of latus rectum: ___________
- Focus: ___________
- Directrix: ___________

3. $2.5y^2 = -20x$



- Length of latus rectum: __________

- Focus: __________

- Directrix: __________

A Second Look at Parabolas

Name: ____ANSWER KEY______
Date: ______________________

Review

1. Yesterday, you learned that $x^2 = 4py$ is the equation of a parabola which opens vertically. How can you tell whether the parabola opens up or down?

   Parabola opens up if p > 0, down if p < 0

2. What does the value *p* represent in the parabola's equation?

   p represents the distance from the vertex to the focus and the directrix

3. The latus rectum is a special segment associated with parabolas. Describe the properties of the latus rectum.

   It connects two points of the parabola and has length 4p.

   It is parallel to the directrix and contains the focus.

Predict

Take the equation of a vertical parabola and switch the *x* and *y*, so that the equation becomes $y^2 = 4px$. Make some predictions about this new equation.

1. How will the graph change?

   Parabola is rotated so that it opens horizontally

2. How will the sign of *p* affect the graph?

   Parabola opens right if p > 0, left if p < 0

3. Will the equation $y^2 = 4px$ represent a function? Explain why or why not.

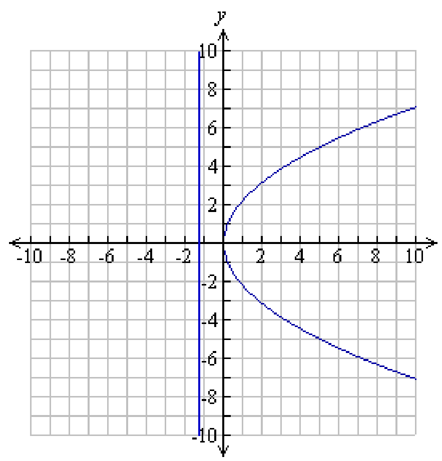
   No, all x-values in the domain (except 0) have two corresponding y-values

   Also, a vertical line through the graph will intersect it at 2 points (except at the vertex)

Apply

Change each equation to standard form, $y^2 = 4px$. Identify the focus, directrix, and length of latus rectum. Then, sketch the graph.

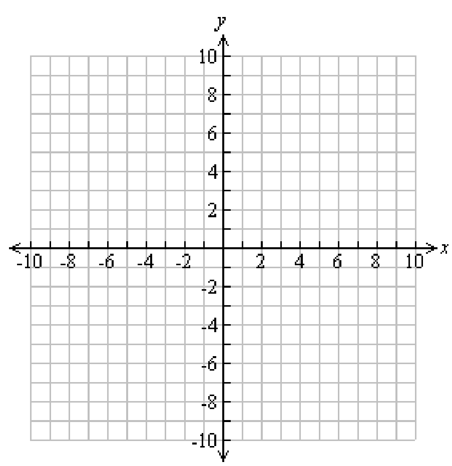
1. $y^2 - 5x = 0$



$y^2 = 5x$, $p = 1.25$

- Length of latus rectum: ____5______
- Focus: _____$(1.25,0)$_____
- Directrix: __$x = -1.25$________

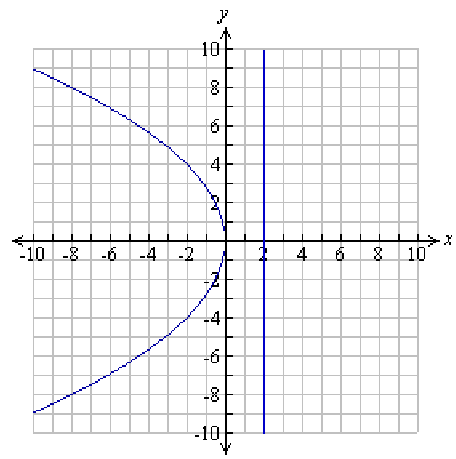
2. $3y^2 = 36x$



$y^2 = 12x$, $p = 3$

- Length of latus rectum: ____12_____
- Focus: ___$(3,0)$_______
- Directrix: ___$x = -3$_______

3. $2.5y^2 = -20x$



$y^2 = -8x$, $p = -2$

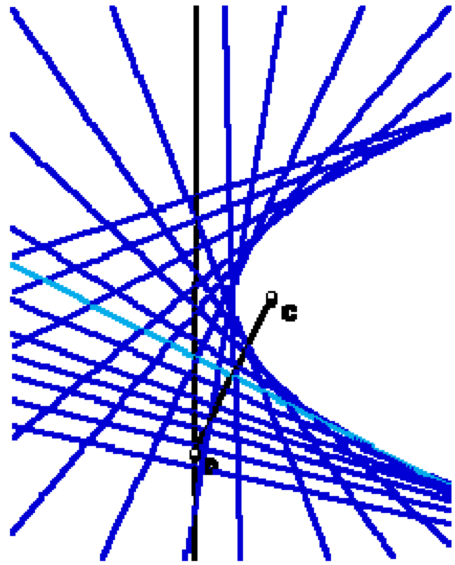
- Length of latus rectum: ___8_____
- Focus: _____$(-2,0)$_____
- Directrix: ___$x = 2$_______

Folding Parabolas

Name: ________________________
Date: _________________________

Using a piece of patty paper and a pencil, follow each step listed below to fold a parabola about a given focus and directrix.

1. On the wax paper, draw a line *l* to represent the directrix and a point *C,* not on *l*, to represent the focus.



2. Fold the paper so that *C* coincides with some point *D* on *l*. Crease the paper well, then unfold.

3. Fold the paper so that *C* coincides with a different point *D* on *l.* Again, crease well, then unfold.

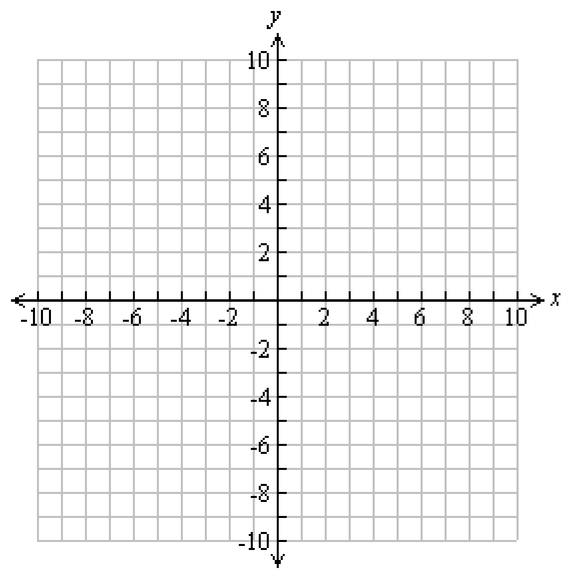
4. Repeat step 3 until you see a curve being traced out by the collection of lines. The more folds made, and the closer the point *D*'s are to each other, the better the results will be.

5. What is the curve formed?

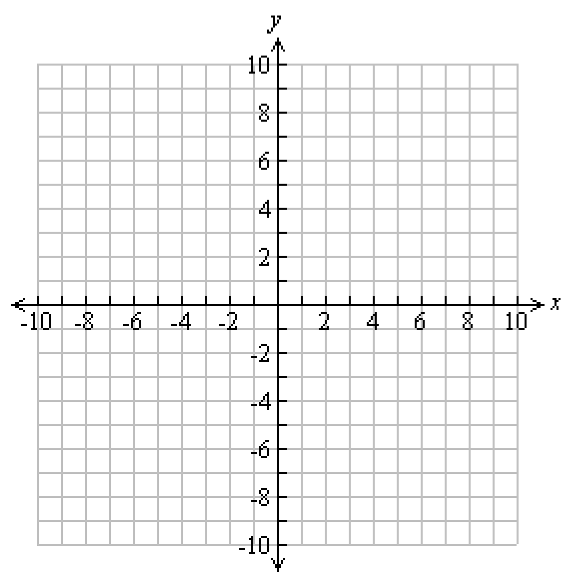
Response Board Practice

Examples:

1. Find the equation of the parabola with directrix $x = 4$.



2. Find the equation of the parabola that represents a function and contains the point (–10, 20).



Now, you try!

Find the equation of the parabola, vertex at the origin, with the given property:

Exercise	Solution
1. The directrix is $y = -8$.	$x^2 = 32y$
2. The focus is (1, 0).	$y^2 = 4x$
3. The latus rectum is 6 units long and lies in quadrants III and IV.	$x^2 = -6y$
4. The focus is $(0, -2.7)$.	$x^2 = -10.8y$

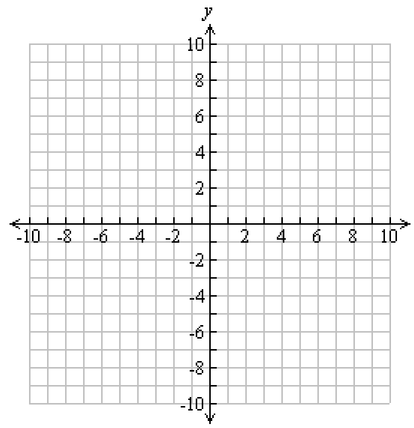
5. The graph is a function and contains the point $(3,-12)$.	$x^2 = -\frac{3}{4}y$
6. The directrix is $x = \frac{10}{7}$.	$y^2 = -\frac{40}{7}x$
7. The graph is not a function and contains the point (2, 6).	$y^2 = 18x$
8. The directrix is the $y-$ axis	???

Plotting Parabolas
Homework

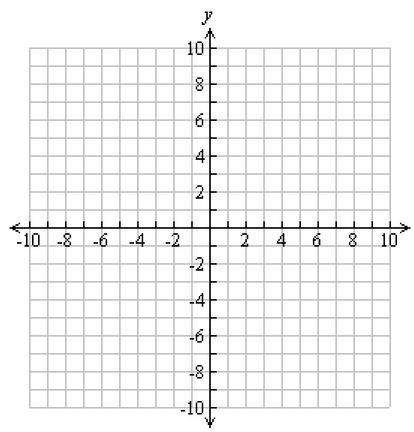
Name: ____________________________
Date: ____________________________

Directions: Change each equation into standard form. Find the focus, directrix, and latus rectum, then sketch the parabola.

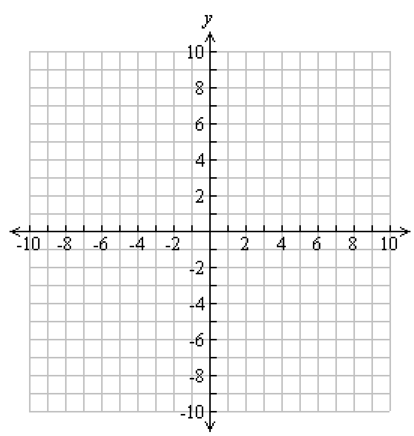
1. $x^2 + 20y = 0$



2. $3y^2 = -12x$

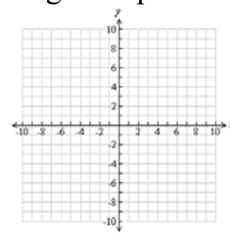


3. $8y^2 - 4x = 0$

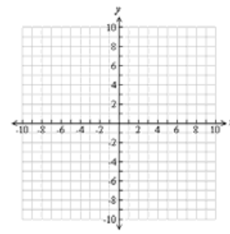


For exercises 4 – 7, determine the equation of the parabola that has vertex at the origin and the given focus or directrix. Begin by sketching each parabola.

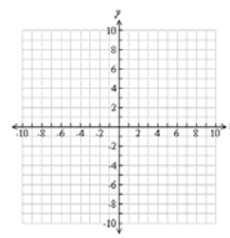
4. Focus is (8, 0)



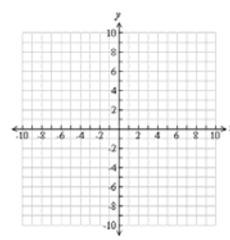
5. Focus is (0, –6.5)



6. Directrix is $x = 9.25$

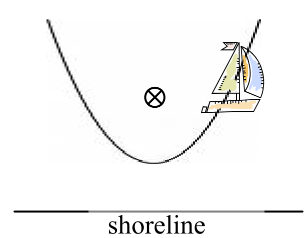


7. Directrix is $y = -\dfrac{20}{3}$



8. If you are given the equation of a parabola with vertex at origin, explain how to determine if the parabola opens to the right, left, up, or down.

9. The course for a sailboat race includes a turnaround point marked by a stationary buoy $\otimes$. The sailboats follow a path that remains equidistant from the buoy and the shoreline. The distance from the buoy to the shoreline is 2.5 miles



a. Find an equation representing the parabolic path of the sailboat.

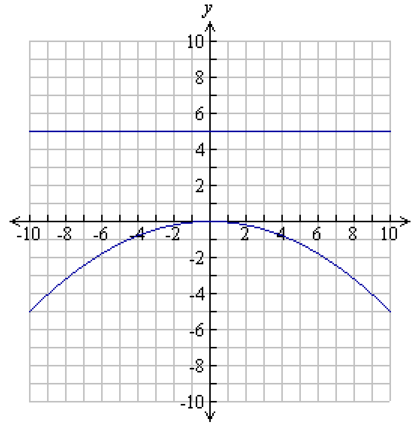
b. At two points along its course, the line connecting the boat to the buoy is parallel to the shoreline. How far is the boat from the buoy at either of these points?

Plotting Parabolas Name: __ANSWER KEY_________
Homework Date: ___________________________

Directions: Change each equation into standard form. Find the focus, directrix, and latus rectum of each parabola. Sketch the graph, changing the scale if necessary.

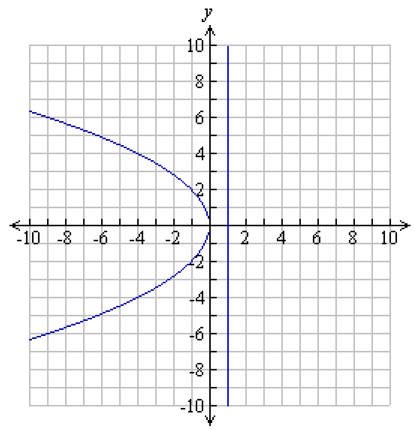
1. $x^2 + 20y = 0$

   $x^2 = -20y$
   $p = -5$, opens down
   Focus (0, –5)
   Directrix $y = 5$
   Latus rectum = 20



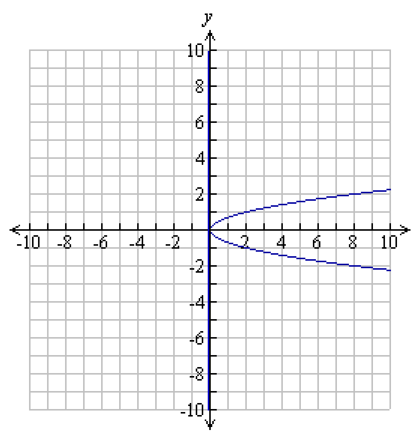
2. $3y^2 = -12x$

   $y^2 = -4x$
   $p = -1$, opens left
   Focus (–1, 0)
   Directrix $x = 1$
   Latus rectum = 4



3. $8y^2 - 4x = 0$

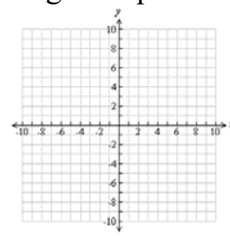
   $y^2 = 0.5x$
   $p = \frac{1}{8}$, opens right
   Focus $\left(\frac{1}{8}, 0\right)$
   Directrix $x = -\frac{1}{8}$
   Latus rectum = 0.5



For exercises 4 – 7, determine the equation of the parabola that has vertex at the origin and the given focus or directrix. Begin by sketching each parabola.

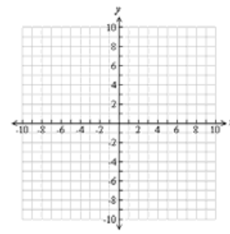
4. Focus is (8, 0)

$y^2 = 32x$



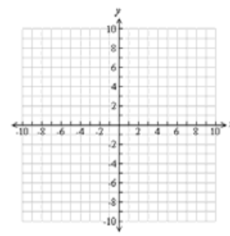
5. Focus is (0, –6.5)

$x^2 = -26y$



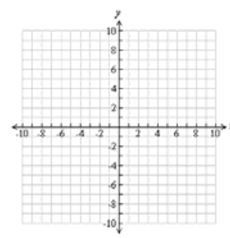
6. Directrix is $x = 9.25$

$y^2 = -37x$



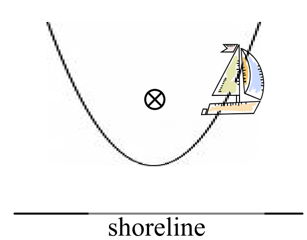
7. Directrix is $y = -\dfrac{20}{3}$

$x^2 = \dfrac{80}{3}y$



8. If you are given the standard form of the equation of a parabola with vertex at origin, explain how to determine if the parabola opens to the right, left, up, or down.
If equation has x^2*, parabola opens vertically, up if* $p > 0$*, down if* $p < 0$*. If equation has* y^2*, parabola opens horizontally, right if* $p > 0$*, left if* $p < 0$*.*

9. The course for a sailboat race includes a turnaround point marked by a stationary buoy ⊗. The sailboats follow a path that remains equidistant from the buoy and the shoreline. The distance from the buoy to the shoreline is 2.5 miles



a. Find an equation representing the parabolic path of the sailboat.
$x^2 = 5y$

b. At two points along its course, the line connecting the boat to the buoy is parallel to the shoreline. How far is the boat from the buoy at either of these points?
These points are the endpoints of the latus rectum, so distance = 2p = 2.5 miles

Polishing Up Parabolas Name: ________________________

Warm Up Date: __________________________

	Condition	Circle One	
1.	Graph is not a function.	$x^2 = 4py$	$y^2 = 4px$
2.	Directrix is a vertical line.	$x^2 = 4py$	$y^2 = 4px$
3.	Focus is on the y – axis.	$x^2 = 4py$	$y^2 = 4px$
4.	Parabola opens vertically.	$x^2 = 4py$	$y^2 = 4px$
5.	Latus rectum is parallel to the x – axis.	$x^2 = 4py$	$y^2 = 4px$

6. Recall from geometry that the equation of a circle centered at the origin with radius r is $x^2 + y^2 = r^2$. How does this equation change if the center of the circle is moved to the point *(h, k)*?

7. In our previous study of quadratics, we wrote the equation of a parabola with vertex at the origin in the form $y = ax^2$. To move the vertex to the point (h, k), how do we modify the equation?

Polishing Up Parabolas Name: ___ANSWER KEY________
Warm Up Date: ____________________________

	Condition	Circle One	
1.	Graph is not a function.	$x^2 = 4py$	$\boxed{y^2 = 4px}$
2.	Directrix is a vertical line.	$x^2 = 4py$	$\boxed{y^2 = 4px}$
3.	Focus is on the y – axis.	$\boxed{x^2 = 4py}$	$y^2 = 4px$
4.	Parabola opens vertically.	$\boxed{x^2 = 4py}$	$y^2 = 4px$
5.	Latus rectum is parallel to the x – axis.	$\boxed{x^2 = 4py}$	$y^2 = 4px$

6. Recall from geometry that the equation of a circle centered at the origin with radius r is $x^2 + y^2 = r^2$. How does this equation change if the center of the circle is moved to the point *(h, k)*?

   The equation becomes $(x-h)^2 + (y-k)^2 = r^2$

7. In our previous study of quadratics, we wrote the equation of a parabola with vertex at the origin in the form $y = ax^2$. To move the vertex to the point (h, k), how do we modify the equation?

   - The form is $y = a(x-h)^2 + k$, or $y - k = a(x-h)^2$
   - Comparing the second form to the original equation $y = ax^2$, x has been replaced with $x - h$ and the y replaced with $y - k$
   - In #6, the center of the circle was moved by replacing x with x-h and y with y-k.
   - Following the same pattern as in these two examples, replacing x with x-h and y with y-k should move the vertex from the origin to (h, k).
     - $y^2 = 4px \rightarrow (y-k)^2 = 4p(x-h)$
     - $x^2 = 4py \rightarrow (x-h)^2 = 4p(y-k)$

Examples for Modeling – Graphing Parabolas with Vertex not at the Origin

Graph & describe the key features (vertex, focus, directrix, latus rectum) of each parabola.

Example 1 **Parabola of the form** $(x-h)^2 = 4p(y-k)$

$(x-1)^2 = 8(y+4)$

- Vertex is (1, -4)
- The expression with x is squared, so parabola opens vertically; p > 0, so it opens up

At this point, teacher should plot the vertex, and lightly sketch a parabola opening up.

- 4p = 8, so p = 2
- Count 2 units up from vertex to determine that the focus is (1, -2); plot & label point F
- Count 2 units down from the vertex to find the directrix is y = -6; draw as dotted line
- Length of latus rectum = 8
- Starting at focus, move 4 units left and 4 units right to plot the endpoints of the latus rectum; use these two points and the vertex to graph the parabola

Example 2 **Parabola of the form** $(y-k)^2 = 4p(x-h)$

$\left(y+\frac{2}{3}\right)^2 = -16(x-2)$

- Vertex is (2, $-\frac{2}{3}$)
- The expression with y is squared, so parabola opens horizontally; p < 0, so it opens left

At this point, teacher should plot the vertex, and lightly sketch a parabola opening left.

- 4p = -16, so p = -4
- Count 4 units left from vertex to determine that the focus is (-2, $-\frac{2}{3}$); plot & label point F
- Count 4 units right from the vertex to find the directrix is x = 6; draw as dotted line
- Length of latus rectum = 16
- Starting at focus, move 8 units up and 8 units down to plot the endpoints of the latus rectum; use these two points and the vertex to graph the parabola

Goin' Round with Parabolas
Round Table One

Person 1: ______________________
Person 2: ______________________
Person 3: ______________________

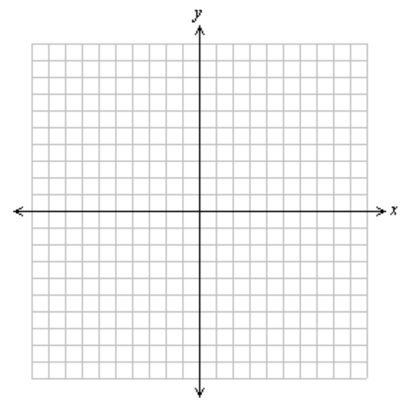
Equation: $(x-2)^2 = -14(y+3)$

Person 1

State the vertex of the parabola: ______________________

Decide which direction the parabola opens: Up Down Left Right

Draw a *rough sketch* of parabola:



Person 2 check and initial:

Person 2

Give the length of the latus rectum: ______________________

Find the focus of the parabola: ______________________

Determine the equation of the directrix: ______________________

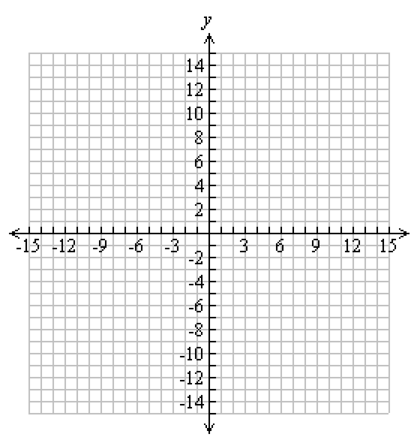
Person 3 check and initial:

Person 3

Draw an accurate graph of the parabola.

Label the focus *F*.

Draw the directrix as a dotted line



Person 1 check and initial:

Goin' Round with Parabolas
Round Table Two

Person 1: ______________________
Person 2: ______________________
Person 3: ______________________

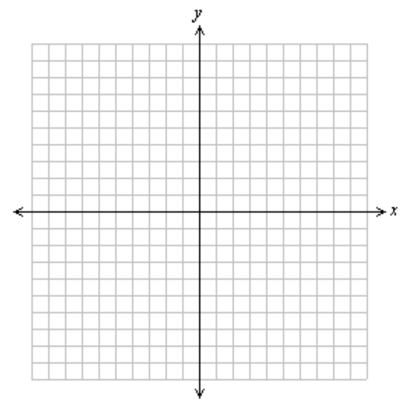
$$\text{Equation: } (y-4)^2 = 6(x+1)$$

Person 1

State the vertex of the parabola: ______________________

Decide which direction the parabola opens: Up Down Left Right

Draw a *rough sketch* of parabola:



Person 2 check and initial:

Person 2

Give the length of the latus rectum: ______________________

Find the focus of the parabola: ______________________

Determine the equation of the directrix: ______________________

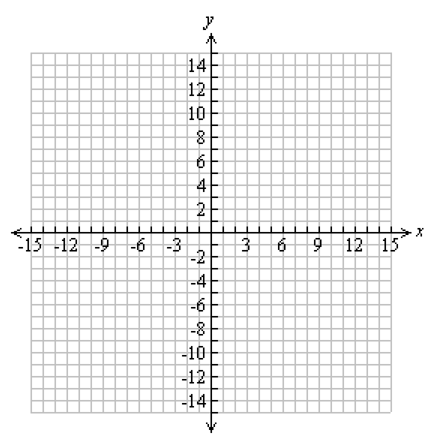
Person 3 check and initial:

Person 3

Draw an accurate graph of the parabola.

Label the focus *F*.

Draw the directrix as a dotted line



Person 1 check and initial:

Goin' Round with Parabolas
Round Table Three

Person 1: ______________________
Person 2: ______________________
Person 3: ______________________

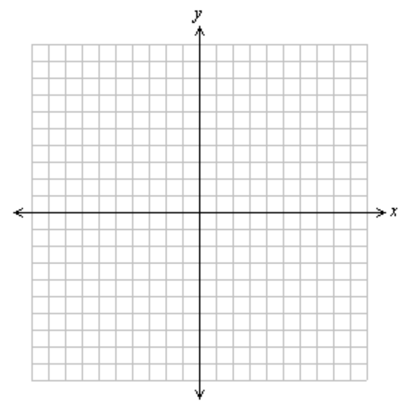
$$\text{Equation: } \left(x+\frac{7}{3}\right)^2 = 12y$$

Person 1

State the vertex of the parabola: ______________________

Decide which direction the parabola opens: Up Down Left Right

Draw a *rough sketch* of parabola:



Person 2 check and initial:

Person 2

Give the length of the latus rectum: ______________________

Find the focus of the parabola: ______________________

Determine the equation of the directrix: ______________________

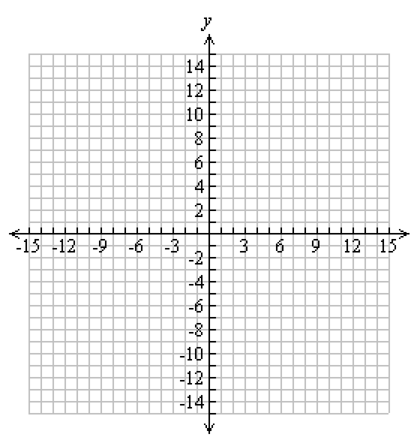
Person 3 check and initial:

Person 3

Draw an accurate graph of the parabola.

Label the focus *F*.

Draw the directrix as a dotted line



Person 1 check and initial:

Goin' Round with Parabolas
Round Table One

Person 1: _ANSWER KEY_______
Person 2: ______________________
Person 3: ______________________

Equation: $(x-2)^2 = -14(y+3)$

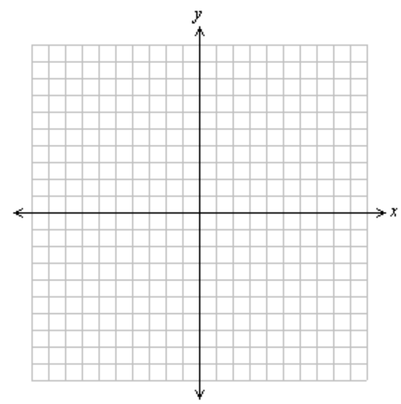
Person 1

State the vertex of the parabola: $(2, -3)$

Decide which direction the parabola opens: Up $\boxed{\text{Down}}$ Left Right

Draw a *rough sketch* of parabola:

See student work.



Person 2 check and initial:

Person 2

Give the length of the latus rectum: 14

Find the focus of the parabola: $(2, -6.5)$

Determine the equation of the directrix: $y = 0.5$

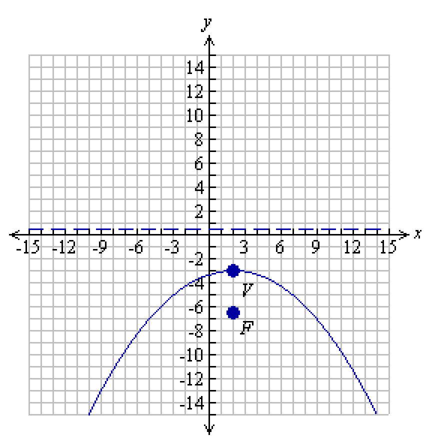
Person 3 check and initial:

Person 3

Draw an accurate graph of the parabola.

Label the focus *F*.

Draw the directrix as a dotted line



Person 1 check and initial:

Goin' Round with Parabolas
Round Table Two

Person 1: _ANSWER KEY_______
Person 2: ______________________
Person 3: ______________________

Equation: $(y-4)^2 = 6(x+1)$

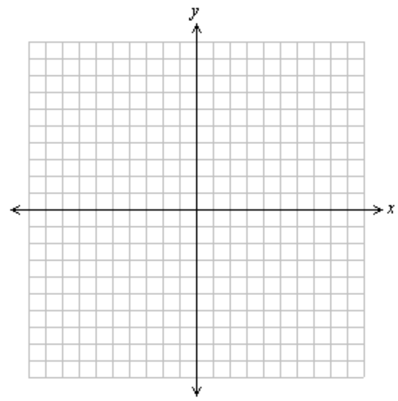
Person 1

State the vertex of the parabola: $(-1,4)$

Decide which direction the parabola opens: Up Down Left $\boxed{\text{Right}}$

Draw a *rough sketch* of parabola:

See student work.



Person 2 check and initial:

Person 2

Give the length of the latus rectum: 6

Find the focus of the parabola: (0.5, 4)

Determine the equation of the directrix: $x = -2.5$

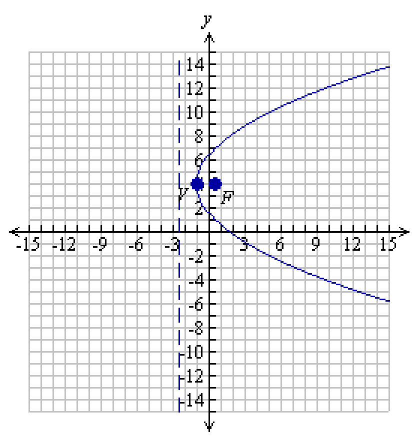
Person 3 check and initial:

Person 3

Draw an accurate graph of the parabola.

Label the focus *F*.

Draw the directrix as a dotted line



Person 1 check and initial:

Goin' Round with Parabolas
Round Table Three

Person 1: _ANSWER KEY________
Person 2: ________________________
Person 3: ________________________

$$\text{Equation: } \left(x+\frac{7}{3}\right)^2 = 12y$$

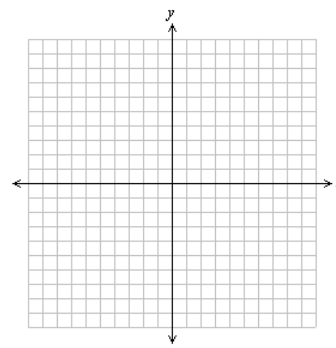
Person 1

State the vertex of the parabola: $\left(-\frac{7}{3}, 0\right)$

Decide which direction the parabola opens: $\boxed{\text{Up}}$ Down Left Right

Draw a *rough sketch* of parabola:

See student work.



Person 2 check and initial:

Person 2

Give the length of the latus rectum: 12

Find the focus of the parabola: $\left(-\frac{7}{3}, 3\right)$

Determine the equation of the directrix: $y = -3$

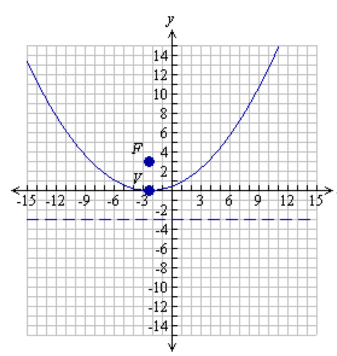
Person 3 check and initial:

Person 3

Draw an accurate graph of the parabola.

Label the focus *F*.

Draw the directrix as a dotted line



Person 1 check and initial:

Examples for Modeling – Writing Equations of Parabolas with Vertex not at the Origin

Determine the equation of the parabola using the given information.

<u>Example 1</u> Vertex is (5, 2), Focus is (0,2)

- Plot the vertex and focus, make a rough sketch of parabola to see it opens left
- Parabola opens horizontally, so form of equation is $(y-k)^2 = 4p(x-h)$
- Distance from vertex to focus is 5 and parabola opens left, so p = -5
- Substitute p = -5 and (h,k) = (5,2) into the equation:
  $(y-2)^2 = -20(x-5)$

<u>Example 2</u> Focus is (-2, 1), Directrix is y = -6

- Plot the focus and directrix, make a rough sketch of parabola to see it opens up
- Parabola opens vertically, so form of equation is $(x-h)^2 = 4p(y-k)$
- To find the vertex, take the midpoint of the vertical segment connecting the focus to the directrix: vertex is (-2, -2.5)
- Distance from vertex to focus is 3.5 and parabola opens up, so p = 3.5
- Substitute p = 3.5 and (h,k) = (-2, -2.5) into the equation:
  $(x+2)^2 = 14(y+2.5)$

Go Around Again!
Round Table One
Writing Equations

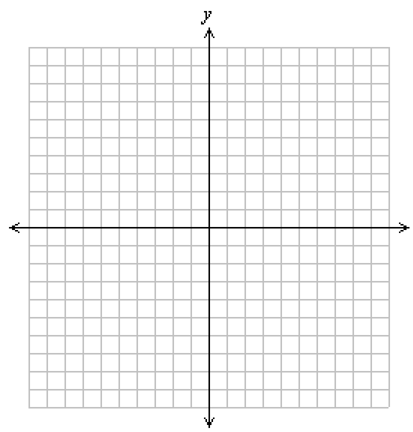
Person 1: ______________________
Person 2: ______________________
Person 3: ______________________

Properties:

- ❖ Focus is (2, 1)
- ❖ Directrix is $x = -4$

Person 1

Graph the properties on the coordinate grid:



Draw a *rough sketch* of parabola:

Person 2 check and initial:

Person 2

Determine the vertex of the parabola: ______________________

Calculate the value of p: ______________________

Person 3 check and initial:

Person 3

Which way does the parabola open? Vertically Horizontally

Choose the correct form of equation: $(y-k)^2 = 4p(x-h)$ $(x-h)^2 = 4p(y-k)$

Substitute the value of p and coordinates of the vertex. Write the final equation. ______________________

Person 1 check and initial:

Go Around Again!
Round Table Two
Writing Equations

Person 1: ______________________
Person 2: ______________________
Person 3: ______________________

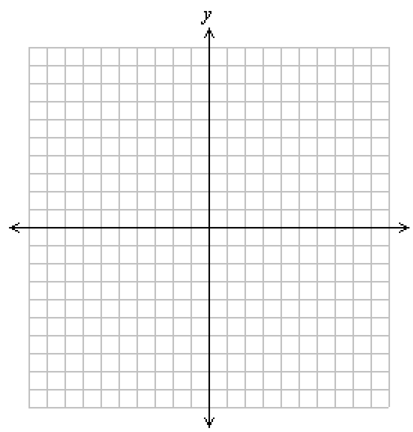
Properties:

- ❖ Vertex is (–3, 5)
- ❖ Focus is (–3, 7)

Person 1

Graph the properties on the coordinate grid:

Draw a *rough sketch* of parabola:



Person 2 check and initial:

Person 2

Calculate the value of p: ______________________

Determine the equation of the directrix: ______________________

Person 3 check and initial:

Person 3

Which way does the parabola open? Vertically Horizontally

Choose the correct form of equation: $(y-k)^2 = 4p(x-h)$ $(x-h)^2 = 4p(y-k)$

Substitute the value of p and coordinates of the vertex. Write the final equation. ______________________

Person 1 check and initial:

Go Around Again!
Round Table Three
Writing Equations

Person 1: ______________________
Person 2: ______________________
Person 3: ______________________

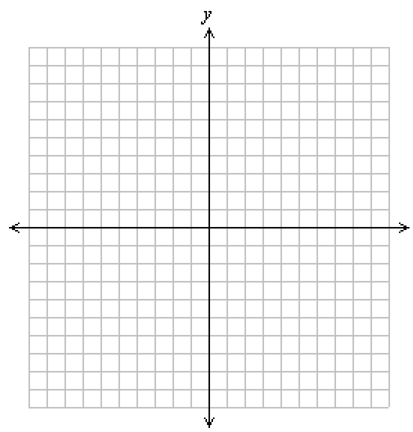
Properties:

- ❖ Vertex is (2.5, –4)
- ❖ Directrix is $x = 7$

Person 1

Graph the properties on the coordinate grid:

Draw a *rough sketch* of parabola:



Person 2 check and initial:

Person 2

Calculate the value of p: ______________________

Determine the coordinates of the focus. ______________________

Person 3 check and initial:

Person 3

Which way does the parabola open? Vertically Horizontally

Choose the correct form of equation: $(y-k)^2 = 4p(x-h)$ $(x-h)^2 = 4p(y-k)$

Substitute the value of p and coordinates of the vertex. Write the final equation. ______________________

Person 1 check and initial:

Go Around Again!
Round Table One
Writing Equations

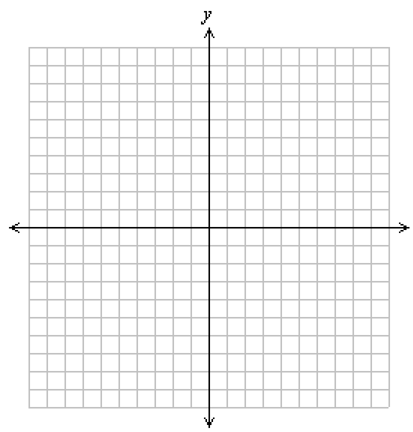
Person 1: ___ANSWER KEY_____
Person 2: ______________________
Person 3: ______________________

Properties:

- ❖ Focus is (2, 1)
- ❖ Directrix is $x = -4$

Person 1

Graph the properties on the coordinate grid:



Draw a *rough sketch* of parabola:
See student work.

Person 2 check and initial:

Person 2

Determine the vertex of the parabola: (–1, 1)

Calculate the value of p: $p = 3$

Person 3 check and initial:

Person 3

Which way does the parabola open? Vertically $\boxed{\text{Horizontally}}$

Choose the correct form of equation: $\boxed{(y-k)^2 = 4p(x-h)}$ $(x-h)^2 = 4p(y-k)$

Substitute the value of p and coordinates of the vertex. Write the final equation. $(y-1)^2 = 12(x+1)$

Person 1 check and initial:

Go Around Again!
Round Table Two
Writing Equations

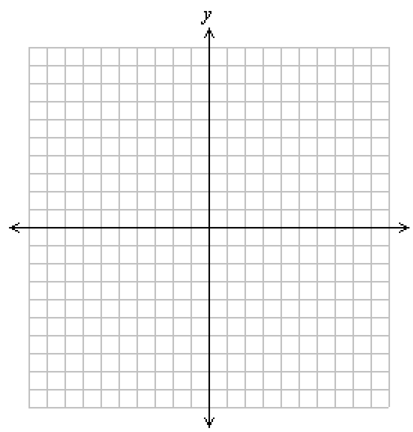
Person 1: ___ANSWER KEY_____
Person 2: ______________________
Person 3: ______________________

Properties:

- ❖ Vertex is (–3, 5)
- ❖ Focus is (–3, 7)

Person 1

Graph the properties on the coordinate grid:



Draw a *rough sketch* of parabola:
See student work.

Person 2 check and initial:

Person 2

Calculate the value of p: $p = 2$

Determine the equation of the directrix: $y = 3$

Person 3 check and initial:

Person 3

Which way does the parabola open? $\boxed{\text{Vertically}}$ Horizontally

Choose the correct form of equation: $(y-k)^2 = 4p(x-h)$ $\boxed{(x-h)^2 = 4p(y-k)}$

Substitute the value of p and coordinates of the vertex. Write the final equation. $(x+3)^2 = 8(y-5)$

Person 1 check and initial:

Go Around Again!
Round Table Three
Writing Equations

Person 1: ___ANSWER KEY_____
Person 2: ______________________
Person 3: ______________________

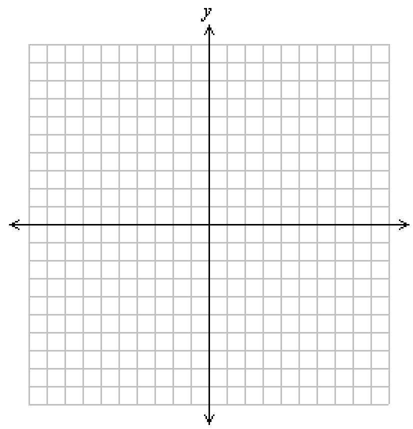
Properties:

- ❖ Vertex is (2.5, –4)
- ❖ Directrix is $x = 7$

Person 1

Graph the properties on the coordinate grid:

Draw a *rough sketch* of parabola:



Person 2 check and initial:

Person 2

Calculate the value of p: $p = 4.5$

Determine the coordinates of the focus. (–2, –4)

Person 3 check and initial:

Person 3

Which way does the parabola open? Vertically $\boxed{\text{Horizontally}}$

Choose the correct form of equation: $\boxed{(y-k)^2 = 4p(x-h)}$ $(x-h)^2 = 4p(y-k)$

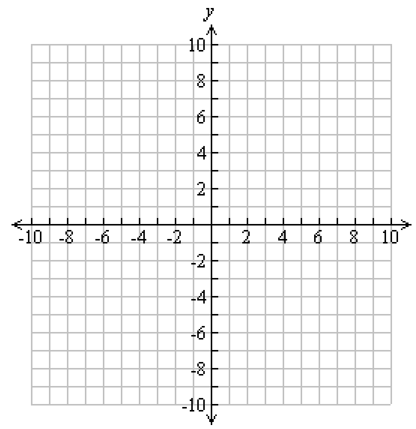
Substitute the value of p and coordinates of the vertex. Write the final equation. $(y+4)^2 = 18(x-2.5)$

Person 1 check and initial:

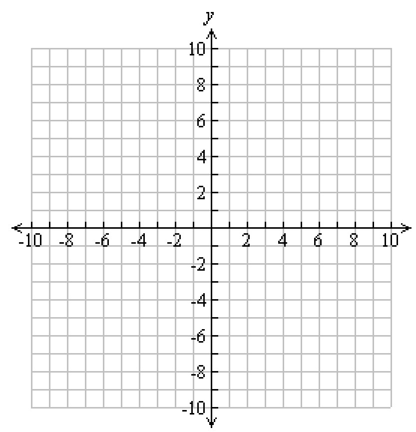
Making a U-Turn Name: ______________________
Homework Date: ______________________

Directions: Determine the vertex, focus, and directrix of each parabola. Give the latus rectum, and then sketch the graph.

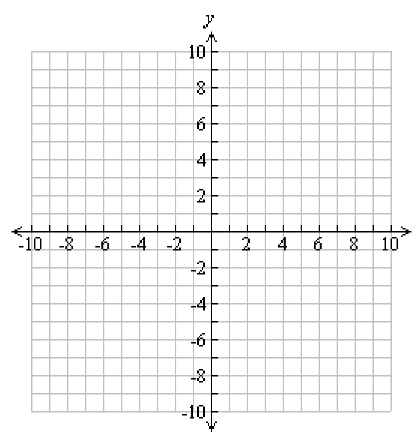
1. $(x+1)^2 = -4(y+1)$



2. $(y-2)^2 = 8(x+3)$



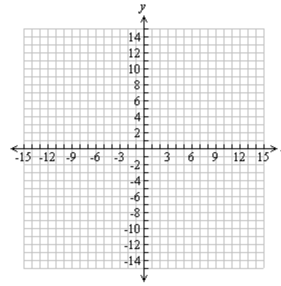
3. $y^2 = -9(x-4.5)$



For exercises 4 – 7, find the standard form of the parabola with the given properties. Begin by sketching each parabola.

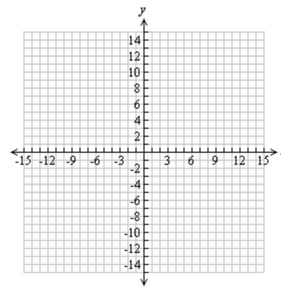
4. Vertex (2, –3)

   Focus (2, –5)



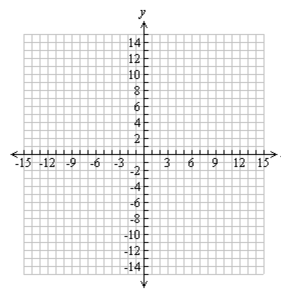
5. Vertex (5, –2)

   Directrix $x = 1.5$



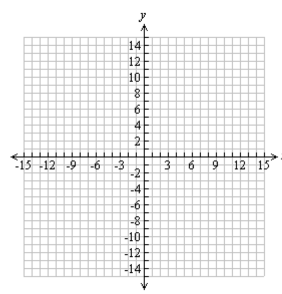
6. Focus (–7, 0)

   Directrix: $x = 11$



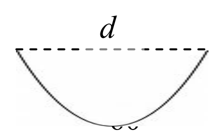
7. Focus (–2, 4)

   Directrix: $y = -9$



8. Describe at least two similarities and two differences between the graphs of $y^2 = 4(x+1)$ and $y^2 = -4(x-1)$.

<u>Challenge</u> Next class, you will learn more about the applications of parabolas and their focus. A certain telescope dish has the shape of a parabola with a focal length (distance from vertex to focus) of 6.8 feet. If the depth of the telescope is 10.2 feet, calculate its diameter, d.



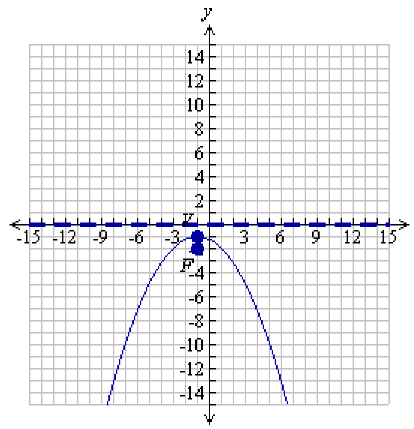
Making a U-Turn
Homework

Name: ___ANSWER KEY________
Date: __________________________

Directions: Determine the vertex, focus, and directrix of each parabola. Give the latus rectum, and then sketch the graph.

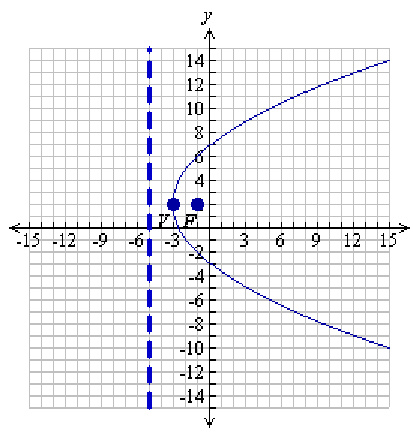
1. $(x+1)^2 = -4(y+1)$

   Vertex (–1, –1)
   Focus (–1, –2)
   Directrix: $y = 0$
   Latus Rectum = 4



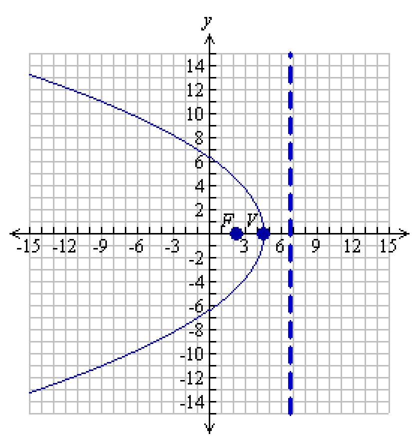
2. $(y-2)^2 = 8(x+3)$

   Vertex (–3, 2)
   Focus (–1, 2)
   Directrix: $x = -5$
   Latus Rectum = 8



3. $y^2 = -9(x-4.5)$

   Vertex (4.5, 0)
   Focus (2.25, 0)
   Directrix: $x = 6.75$
   Latus Rectum = 9

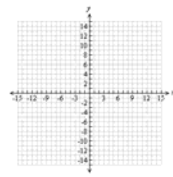


For exercises 4 – 7, find the standard form of the parabola with the given properties. Begin by sketching each parabola.

4. Vertex (2, –3)

   Focus (2, –5)

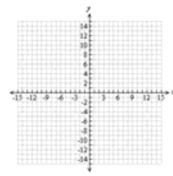
   $(x-2)^2 = -8(y+3)$



5. Vertex (5, –2)

   Directrix $x = 1.5$

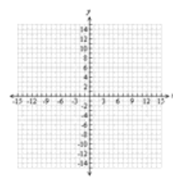
   $(y+2)^2 = 14(x-5)$



6. Focus (–7, 0)

   Directrix: $x = 11$

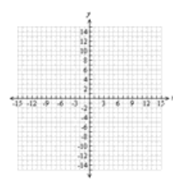
   $y^2 = -36(x-2)$



7. Focus (–2, 4)

   Directrix: $y = -9$

   $(x+2)^2 = 26(y+2.5)$

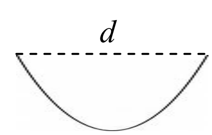


8. Describe at least two similarities and two differences between the graphs of $y^2 = 4(x+1)$ and $y^2 = -4(x-1)$.

   Answers may vary:
   Similarities: same focus (origin), same focal length (1), both open horizontally, neither are functions, same axis of symmetry (x–axis)
   Differences: different vertex, open in opposite directions, directrices are different
   Note that the graphs are reflections of each other over the y–axis

9. In an upcoming activity, you will learn more about the applications of parabolas and their focus. A certain telescope dish has the shape of a parabola with a focal length (distance from vertex to focus) of 6.8 feet. If the depth of the telescope is 10.2 feet, calculate its diameter, d.

   Diameter ≈ 33.3 feet



Examples for Reteaching & Review

1. Problem: Find the focus and directrix of the parabola given by $y^2 = 12x$, then graph it.
   - $y^2 = 4px \rightarrow 4p = 12 \rightarrow p = 3$
   - Because $p>0$, the parabola, with its x –axis symmetry, opens to the right.
   - Focus: The focus is 3 units to the right of the vertex, so $(p,0) = (3,0)$
   - Directrix: $x = -p$, so $x = -3$ (draw on the graph)
   - To graph the parabola, we will use the endpoints of the latus rectum. Its length is 12, so the points 6 units above and 6 units below the focus lie on the parabola: (3,6) and (3,-6). Use these two points and the vertex to graph the parabola.

2. Problem: Find the focus and directrix of the parabola given by $x^2 = -8y$, then graph it.
   - Here $4p = -8$; $p=-2$, $p<0$, the parabola, with its y- axis symmetry, opens downward. The focus is 2 units below the vertex, so (p, 0) = (-2, 0)
   - To graph the parabola, we will use the vertex, (0,0), and the two endpoints of the latus rectum. The length of the latus rectum is $|4p| = 8$.
   - Because the graph has y-axis symmetry, the latus extends 4 units to the left and 4 units to the right of the focus,(0, -2). The endpoints of the latus rectum are (-4, -2) and (4, -2). Sketch the parabola through the vertex and these two points.

3. Problem: Find the equation of a parabola with focus (5, 0) and directrix $x = -7$.
   - Vertex is the midpoint of the segment joining the directrix & focus, so (h, k) = (-1, 0).
   - Distance from vertex to focus is 6, so $p = 6$. Parabola opens right, so the equation will be $(y-k)^2 = 4p(x-h)$.
   - Substituting, equation is $y^2 = 4(x+1)$.

4. Problem: Find the vertex, focus, and directrix of the parabola given by $(x-3)^2 = 8(y+1)$.
   - Here $h = 3$; $k = -1$. The vertex of the parabola is (3,-1).
   - $4p = 8$; $p = 2$
   - Focus $(h, k+p) = (3, -1+3) = (3, 1)$
   - Directrix $y = k-p = -1-2 = -3$
   - To graph the parabola, we use the vertex, (3, -1), and the two endpoints of the latus rectum. The length of the latus rectum is $|4p| = 8$
   - Because the graph has vertical symmetry, the latus rectum extends 4 units to the left and 4units to the right of the focus. The endpoints of the latus rectum are (3-4, 1), or (-1, 1), and (3+4, 1), or (7,1). Sketch the parabola using these two points and the vertex.

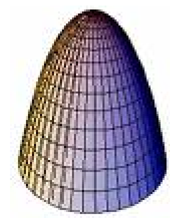
Parabolas Name: ________________________

Extension and Challenge Problems Date: _________________________

1. Find the equations of two parabolas that have vertex (3, 2) and contain the point (7, 4).

2. A sprinkler system shoots a stream of water that follows a parabolic path. The water reaches a maximum height of 20 feet at a horizontal distance of 45 feet from the nozzle. If the nozzle is located at the origin, find an equation describing the water's path.

When a parabola is rotated about its axis of symmetry, a surface called a paraboloid is formed. The shape resembles the nose cone of a rocket. The paraboloid has many applications, some of which you will learn about in our next activity. Here are a few problems involving paraboloids.



3. The formula for the volume of a paraboloid is $V = 0.5\pi r^2 h$, where r is the radius of the circular base and h is the height. What is the formula for the volume of a cone? How do the volume of a cone and paraboloid compare? Explain.

4. The paraboloid nose cone on a model rocket has a volume of 6.6 cm^3. The radius of the nose cone is 1.2 cm. Calculate the height of the nose cone.

5. Rock salt can be used to treat icy surfaces in wintertime. A 21 foot high storage building for rock salt has the shape of a paraboloid. The cross sections of the building are parabolas with a focal length (distance from vertex to focus) of 3.5 ft. Calculate the volume of the building.

Super Challenge

The line $y = 0.5x + 3$ intersects the parabola $y = x^2$ at two points. A triangle is formed using these two points as vertices, along with a third point located on the parabola below the line. What is the maximum possible area of this triangle?

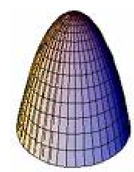
Parabolas Name: __ANSWER KEY_________
Extension and Challenge Problems Date: ________________________

1. Find the equations of two parabolas that have vertex (3, 2) and contain the point (7, 4).

$(x-3)^2 = 8(y-2)$ $(y-2)^2 = (x-3)$

2. A sprinkler system shoots a stream of water that follows a parabolic path. The water reaches a maximum height of 20 feet at a horizontal distance of 45 feet from the nozzle. If the nozzle is located at the origin, find an equation describing the water's path.

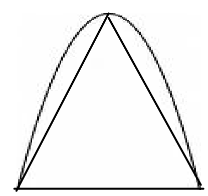
$(x-45)^2 = -101.25(y-20)$



When a parabola is rotated about its axis of symmetry, a surface called a paraboloid is formed. The shape resembles the nose cone of a rocket. The paraboloid has many applications, some of which you will learn about in our next activity. Here are a few problems involving paraboloids.

3. The formula for the volume of a paraboloid is $V = 0.5\pi r^2 h$, where r is the radius of the circular base and h is the height. What is the formula for the volume of a cone? How do the volume of a cone and paraboloid compare? Explain.

$V_{Cone} = \frac{1}{3}\pi r^2 h$. The cross section of a paraboloid is a portion of a parabola; the cross section of a cone is a triangle. The parabola has a slightly larger area than the area of its inscribed triangle. Thus the coefficient in the paraboloid volume formula should be a little larger than $\frac{1}{3}$, which it is, $\frac{1}{2}$.



4. The paraboloid nose cone on a model rocket has a volume of 6.6 cm^3. The radius of the nose cone is 1.2 cm. Calculate the height of the nose cone.

h ≈ 2.9 cm

5. Rock salt can be used to treat icy surfaces in wintertime. A 21 foot high storage building for rock salt has the shape of a paraboloid. The cross sections of the building are parabolas with a focal length (distance from vertex to focus) of 3.5 ft. Calculate the volume of the building.

V ≈ 9698 ft^3

Super Challenge

The line $y = 0.5x + 3$ intersects the parabola $y = x^2$ at two points. A triangle is formed using these two points as vertices, along with a third point located on the parabola below the line. What is the maximum possible area of this triangle?

The line and parabola intersect at (2, 4) and (–1.5, 2.25). Call the coordinates of third vertex (x, x^2). *Use the triangle area determinant formula to get an expression for the area:* $A = -1.75x^2 + 0.875x + 5.25$. *This is a quadratic function opening down, so find its vertex, which is also its maximum point: (0.25, 5.359375). Thus the triangle's maximum area is about 5.36 square units.*

Web Investigation Name: ____________________________

Focusing on Parabolas Date: _____________________________

Parabolas have many real–world applications. What are some you already know about? Today, you will be using the Internet to discover a key property of parabolas, then research applications that relate to that property.

A. A Property of Parabolas

Locate the site www.worsleyschool.net/science/files/parabola/focus.html

1. This site displays a parabola that opens horizontally. The website gives the equation of this graph as $x = ay^2$, while we have been writing such equations in the form $y^2 = 4px$. If these two equations represent the same parabola, write an equation relating *a* and *p*.

2. The graph on this site represents a reflective parabolic surface. Hitting the Beam button simulates a set of parallel light rays striking the parabola. Try the Beam feature for different widths of parabola. Sketch an example, then describe the behavior of the light rays.

3. As the parabola gets wider, how does the focus move?

4. If you could widen the parabola to a vertical line, where would the focus be?

5. If the focus were placed at the vertex, what would be the equation of this graph? What would the shape of the graph be?

6. Another site that nicely illustrates the property from #2 is www.intmath.com/Plane–analytic–geometry/4_Parabola.php. Visit this site and try its parabolic reflector simulation.

7. How accurately can you locate the focus of a parabola, given its graph? Try the parabola–focus game at www.its.caltech.edu/~mamikon/Parafocus.html.

B. Applications of the Parabola and its Focus

As you saw in Part A, when parallel light rays strike a reflective parabolic surface, the reflected beams all pass through a single point, the focus. The converse is also true; if a light source is placed at the focus, the reflected beams emanating from the parabola will be parallel. This leads to many useful applications of the parabola. Your task is to discover and describe three of these applications. For each application, write a short paragraph or series of bullets, which should include:

- √ A description of the application
- √ Why the parabolic shape and focus is important
- √ History/background info on the application

- Application #1

- Application #2

- Application #3

Web Investigation Name: ___ANSWER KEY________
Focusing on Parabolas Date: ________________________

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$$\frac{1}{a} = 4p$$

2. The graph on this site represents a reflective parabolic surface. Hitting the Beam button simulates a set of parallel light rays striking the parabola. Try the Beam feature for different widths of parabola. Sketch an example, then describe the behavior of the light rays.

After reflecting off the parabolic surface, the beams all pass through a common point, the focus.

3. As the parabola gets wider, how does the focus move?

The focus gets farther from the vertex.

4. If you could widen the parabola to a vertical line, where would the focus be?

Infinitely far from the vertex. (There really isn't one, the reflected beams would stay parallel.)

5. If the focus were placed at the vertex, what would be the equation of this graph? What would the shape of the graph be?

If $p = 0$ in $y^2 = 4px$, then $y^2 = 0 \to y = 0$, which is a horizontal line

6. Another site that nicely illustrates the property from #2 is www.intmath.com/Plane–analytic–geometry/4_Parabola.php. Visit this site and try its parabolic reflector simulation.

7. Just for fun! How accurately can you locate the focus of a parabola, given its graph? Try the parabola–focus game at www.its.caltech.edu/~mamikon/Parafocus.html.

Focusing on Parabolas - Quiz

Name: ________________________
Date: _________________________

1. What is a parabola? Use the terms focus and directrix in your definition.

2. Find the equation of the parabola with vertex at the origin and directrix x = -3.

[A] $y^2 = 12x$ [B] $y^2 = -12x$ [C] $x^2 = 12y$ [D] $x^2 = -12y$

3. Find the vertex and focus of the graph of $-8(x-1) = (y+1)^2$

[A] Vertex: (-1, 1), Focus: (1, 1)

[B] Vertex: (-1, 1), Focus: (-3, 1)

[C] Vertex: (1, -1), Focus: (3, -1)

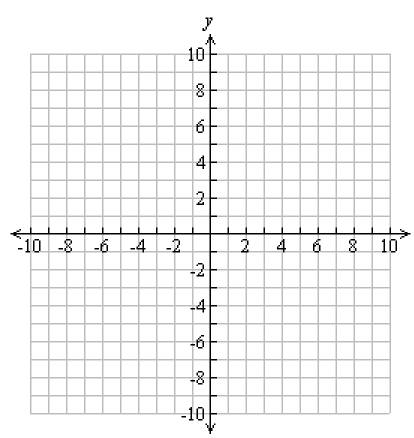
[D] Vertex: (1,-1), Focus: (-1,-1)

4. Determine the equation of the directrix of the parabola $(x-5)^2 = 10(y-2)$.

[A] y = 4.5 [B] y = -0.5 [C] x = 4.5 [D] x = -0.5

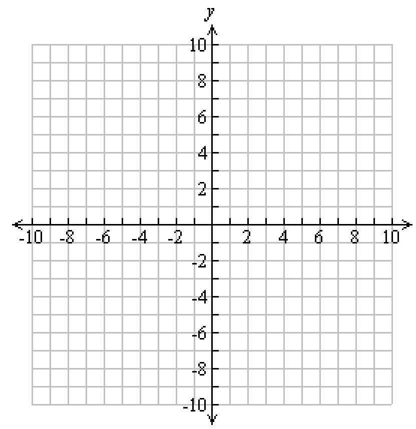
5. Identify the components of the parabola $(x-1)^2 = -16(y-5)$, then graph it.

- Vertex: ________
- Focus: ________
- Directrix: ________
- Length of Latus Rectum: ________



6. Identify the components of the parabola $(y-3)^2 = 7(x+2)$, then graph it.

- ❖ Vertex: _________
- ❖ Focus: _________
- ❖ Directrix: _________
- ❖ Length of Latus Rectum: _________



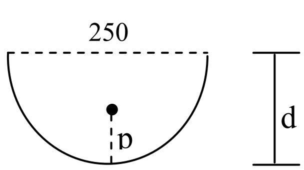
7. Determine the equation of the parabola with the given properties.

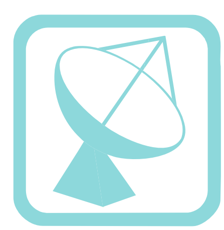
   a. Vertex is (–4, –9), directrix is $y = -2$

   b. Vertex is origin, focus is 1.7 units to the left of the vertex

   c. Directrix is $y = 3$, focus is $\left(\frac{4}{3}, 11\right)$

8. One of the largest radio telescopes has a diameter of 250 feet and a focal length, p, of 50 feet. If the cross section of the telescope is a parabola, find its depth, d.





Focusing on Parabolas - Quiz

Name: ___ANSWER KEY________
Date: __________________________

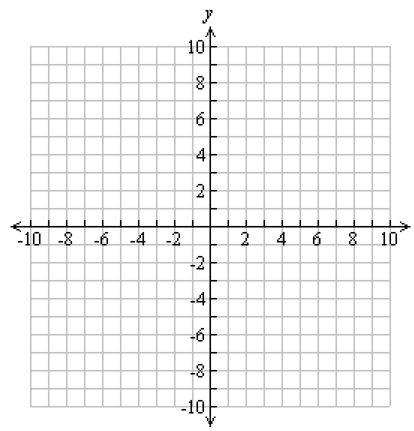
1. What is a parabola? Use the terms focus and directrix in your definition.

   <u>A parabola is the set of points in a plane equidistant from the focus, a given point, and the directrix, a given line</u>

2. **A** 3. **D** 4. **B**

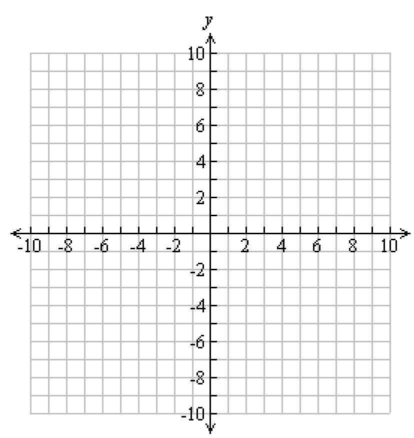
5. Identify the components of the parabola $(x-1)^2 = -16(y-5)$, then graph it.

   - Vertex: _(1, 5)____
   - Focus: __(1, 1)__
   - Directrix: __$y = 9$__
   - Length of Latus Rectum: ___16_____



6. Identify the components of the parabola $(y-3)^2 = 7(x+2)$, then graph it.

   - Vertex: _(–2, 3)____
   - Focus: __(–.25, 3)__
   - Directrix: __$x = -3.75$__
   - Length of Latus Rectum: ___7_____



7. Determine the equation of the parabola with the given properties.

   a. Vertex is (–4, –9), directrix is $y = -2$

      $$(x+4)^2 = -28(y+9)$$

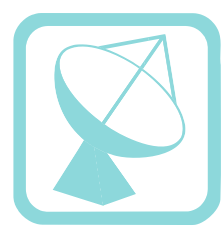
   b. Vertex is origin, focus is 1.7 units to the left of the vertex

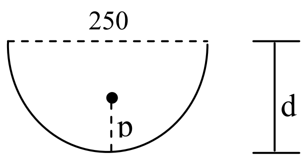
      $$y^2 = -6.8x$$

   c. Directrix is $y = 3$, focus is $\left(\frac{4}{3}, 11\right)$

      $$\left(x - \frac{4}{3}\right)^2 = 16(y-7)$$

8. One of the largest radio telescopes has a diameter of 250 feet and a focal length, p, of 50 feet. If the cross section of the telescope is a parabola, find its depth, d.





Let the vertex be the origin; $x^2 = 4py$ *and* $p = 50$*, so equation of parabola is* $x^2 = 200y$.

The point (125, d) is the upper right point of the telescope. Substituting, $125^2 = 200d$*, so* $d = 78.125$ *feet.*